Mathematical Aspects of Geophysical Flows (MathGeo)

October 12-13, 2023 at IAC-CNR, via dei Taurini 19, 00185 Rome

Schedule

	Thursday 12	Friday 13
10.30 am - 11.30 am	Roberto Natalini	Riccardo Montalto
11.30 am - 12 pm	break	break
12pm - 1 pm	Lucas Ertzbischoff	Augusto del Zotto
2.30 pm - 3.30 pm	Laura Spinolo	Angeliki Menegaki
3.30 pm - 4 pm	break	break
4pm - 5pm	Greg Pavliotis	Andrea Scagliarini

Titles and abstracts

- **The role of oscillations in the 3D Boussinesq equations around a stably stratified Couette flow**
  Speaker: **Augusto Del Zotto** (Imperial College London)
  Abstract: The Boussinesq equations are a fundamental tool for describing the behavior of stratified fluids. When we consider a stable linear stratification combined with a background Couette flow, it gives rise to oscillations, which significantly contribute to the stability of the steady state. In this presentation, we will discuss, at a linear level, how this oscillating structure can be used to achieve enhanced dissipation. Additionally, we will address that these oscillations effectively remove a well-known 3D instability phenomenon referred to as the 'lift-up effect'. This is joint work with Michele Coti Zelati.

- **On thick spray equations**
  Speaker: **Lucas Ertzbischoff** (Imperial College London)
  Abstract: We consider a coupled system between kinetic and fluid equations, describing a cloud of particles immersed within a gas. In the "thick spray" regime, the volume fraction for the particles is not negligible compared to that of the fluid: this raises many difficulties for the study of such system, which seems to present losses of derivatives. In particular,

and contrary to some other fluid-kinetic couplings, its mathematical study has almost remained absent. I will review some recent progress on thick spray equations, and show that one can actually build a Cauchy theory in Sobolev regularity (at least for a compressible viscous fluid) when the initial data satisfies a Penrose type stability condition (being in fact necessary and sufficient for well-posedness).

- **L^2-stability for the 4-waves kinetic equation around the Rayleigh-Jeans equilibrium**
  Speaker: **Angeliki Menegaki** (Imperial College London)
  We consider the four-waves spatial homogeneous kinetic equation arising in weak wave turbulence theory. In this talk I will present some new results on the existence and long-time behaviour of solutions around the Rayleigh-Jeans thermodynamic equilibrium solutions. In particular, introducing a cut-off on the frequencies, I will present an L^2 stability of mild solutions for initial data close to Rayleigh-Jeans, when the dispersion relation is weakly perturbed around the quadratic one. If time permits, I will discuss a more recent result (joint work with Miguel Escobedo) on stability of radial solutions without the cut-off on the frequencies.

- **Nonlinear quasi-periodic oscillations in Euler equations**
  Speaker: **Riccardo Montalto** (Università Statale di Milano)
  In this talk I will present some recent results concerning the construction of quasi-periodic nonlinear waves for the Euler equation bifurcating from some equilibrium solutions (such as constant velocity fields or couette flows). The main technincal ingredients come from KAM (Kolmogorov-Arnold-Moser), Nash Moser and Normal form theory for quasi-linear PDEs and they are also based on microlocal analysis and pseudo-differential operators theory.

- **Mean-fields limits and the multiscale approach to cell movements**
  Speaker: **Roberto Natalini** (IAC-CNR Rome)
  Abstract: In this talk I focus on a quite general class of hybrid mathematical models of collective motions of cells under the influence of chemical stimuli. The models are hybrid in the sense that cells are discrete particles driven by ODE, while the chemoattractant is consid- ered as a continuous signal which solves a diffusive equation. For these models it is possible to prove the mean-field limit in the Wasserstein distance to a system given by the coupling of a Vlasov-type equation with the chemoattractant equation. This approach and results are not based on empirical measures, but rather on marginals of large number of individual densities, and we show the limit with explicit bounds, by proving also existence and uniqueness for the limit system. In the monokinetic case we derive new pressureless nonlocal Euler-type model with chemotaxis, which will be compared with other macroscopic models of cell movement. These results have been obtained in collaboration with Thierry Paul and, for the numerical part, with Marta Menci and Tommaso Tenna.

- **Phase transitions, logarithmic Sobolev inequalities, and uniform-in-time propagation of chaos for weakly interacting diffusions**
  Speaker: **Greg Pavliotis** (Imperial College London)
  Abstract: In this talk I will present recent results on the the mean field limit of weakly interacting diffusions for confining and interaction potentials that are not necessarily convex. We are particularly interested in the relationship between the large N limit of the constant in the logarithmic Sobolev inequality (LSI) for the N-particle system and the presence or absence of phase transitions for the mean field limit. We show that the non-degeneracy of the LSI constant implies uniform-in-time propagation of chaos and Gaussianity of the fluctuations at equilibrium. We also discuss about the consequences of the presence of phase transitions on the combined diffusive/mean field limit of weakly interacting diffusions with a periodic interaction potential.

- **Rayleigh-Taylor turbulence with singular non-uniform initial conditions**
  Speaker: **Andrea Scagliarini** (IAC-CNR Rome)
  Abstract: We will present theoretical and computational results on the phenomenon of Rayleigh-Taylor turbulence with a nonuniform singular initial temperature background. In such conditions, the long-time growth of the mixing layer is still self-similar, but no longer quadratic and depends on the singularity exponent of the initial profile. It will be shown that universality is recovered when looking at the efficiency, defined as the ratio of the variation rates of the kinetic energy over the heat flux. A closure model is proposed that is able to reproduce analytically the time evolution of the mean temperature profiles, in excellent agreement with the numerical results. These findings can be reinterpreted, in the framework of shell models, under the light of *spontaneous stochasticity* where the growth of the mixing layer is mapped into the propagation of a wave of turbulent fluctuations on a rough background. Finally, some results on the non-Boussinesq, stratified case will be also reviewed.

- **Vanishing physical viscosity solutions of characteristic initial-boundary value problems for systems of conservation laws**
  Speaker: **Laura Spinolo** (IMATI-CNR Pavia)
  I will focus on initial-boundary value problems for strictly hyperbolic, nonlinear systems of conservation laws in one space variable and small total variation regimes. It is known that, in general, different viscous mechanisms yield different solutions in the vanishing viscosity limit. I will discuss a new wave front-tracking algorithm providing global-in-time existence of admissible solutions consistent with a broad class of different viscous approximations. The hypotheses cover the most interesting, albeit technically demanding, cases, that is physical (mixed hyperbolic-parabolic) viscosity and characteristic boundary. The result applies to the Euler and inviscid MHD equations, written in both Eulerian and La-

grangian variables. This is joint work with Fabio Ancona and Andrea Marson.

For any question, please contact the organizers:
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